JCH Mathematics - Algebra

Algebra

Algebra and Applied Algebra represents 50% of Paper 1. For basic algebra, there are only a few action verbs we need to recognize

1. **Value** - substitute in value
2. **Multiply out and Simplify**
3. **Factorise** - put in brackets
4. **Solve**
5. **Chart or Draw**

We need to be aware of the following terms

Algebraic Expression - no equal sign
Algebraic Equation - an equal sign

There are a number of things we also need

Inequalities
Simultaneous Equations
Indices
Number Systems
Pythagoras' Theorem
Applied Algebra
Functions

We will review all today

Beware: Not all Algebra

Paper 1

50% Algebra

But

Other 50%:

Numbers, Inequalities, Fractions, Ratios, Sets, Venn Diagrams, Speed and Distance, Indices, Patterns, Financial Mathematics, SURDs

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1 Write each of the following in the form 2^n, where $n \in \mathbb{Q}$.

(a) $2^3 \times 2^5 \times 2^{10}$

$2^3 \times 2^5 \times 2^{10} = 2^8 \times 2^{10}$

$= 2^{18}$

(b) 8^{25}

$8^{25} = (2^3)^{25} = 2^{75}$.

(c) $\sqrt{8}$

$\sqrt{8} = 8^{1/2} = (2^3)^{1/2} = 2^{3/2}$.

2 **(a)** Write each of the following numbers in the form 3^k, where $k \in \mathbb{Q}$.

(i) 9

$9 = 3^2$, so $k = 2$.

(ii) 1

$3^0 = 1$, $k = 1$

(iii) $\sqrt{27}$

$\sqrt{27} = \sqrt{3^3} = (3^3)^{1/2} = 3^{3/2}$ so $k = 3/2$

(iv) $\frac{1}{\sqrt[3]{3}}$

$\frac{1}{\sqrt[3]{3}} = \frac{1}{3^{1/3}} = 3^{-1/3}$, $k = -1/3$.

(b) Write $(-2n)^4$ in the form an^b, where $a, b \in \mathbb{Z}$.

$(-2n)^4 = (-2)^4 \times (n)^4 = 16n^4$

(c) x and $\sqrt{x^2}$ are **not** always equal.

Give an example of a value of x, and the corresponding value of $\sqrt{x^2}$, which are **not** equal.

$x = -2$ $x^2 = 4$ $\sqrt{x^2} = \sqrt{4} = 2$.

$x =$ -2 $\sqrt{x^2} =$ 2.

Factorize Fully - Solved Problems

3 **a** $9x^2 - 81$

$$\begin{aligned} 9x^2 - 81 &= 9(x^2 - 9) \\ &= 9(x^2 - 3^2) \\ &= 9(x - 3)(x + 3) \end{aligned}$$

b $3x^2 - 17x + 10$

❶ $3x^2 - 17x + 10 = (3x - 2)(x - 5)$

or

❷
$$\begin{aligned} 3x^2 - 17x + 10 &= 3x^2 - 15x - 2x + 10 \\ &= 3x(x - 5) - 2(x - 5) \\ &= (3x - 2)(x - 5) \end{aligned}$$

or

❸ $3x^2 - 17x + 10$:

	x	-5
$3x$	$3x^2$	$-15x$
-2	$-2x$	10

$\Rightarrow 3x^2 - 17x + 10 = (x - 5)(3x - 2)$

c $4a - 6bc + 3ac - 8b.$

❶
$$\begin{aligned} 4a - 6bc + 3ac - 8b &= 4a + 3ac - 8b - 6bc \\ &= a(4 + 3c) - 2b(4 + 3c) \\ &= (a - 2b)(4 + 3c) \end{aligned}$$

or

❷
$$\begin{aligned} 4a - 6bc + 3ac - 8b &= 4a - 8b + 3ac - 6bc \\ &= 4(a - 2b) + 3c(a - 2b) \\ &= (4 + 3c)(a - 2b) \end{aligned}$$

d Write the following as a single fraction in its simplest form.

$$\frac{x+2}{3} - \frac{x-1}{2}$$

$$\begin{aligned} \frac{x+2}{3} - \frac{x-1}{2} &= \frac{2(x+2) - 3(x-1)}{6} \\ &= \frac{2x + 4 - 3x + 3}{6} \\ &= \frac{7 - x}{6} \end{aligned}$$

Link to Solve

4 **e** Hence, or otherwise, solve the equation

$$\frac{x+2}{3} - \frac{x-1}{2} = 6.$$

$$\begin{aligned} \frac{x+2}{3} - \frac{x-1}{2} &= \frac{7-x}{6} \\ \Rightarrow \frac{7-x}{6} &= 6 \\ \Rightarrow 7 - x &= 36 \\ \Rightarrow -x &= -29 \\ \Rightarrow x &= 29 \end{aligned}$$

Exercises - Tough Pre-Project Questions

5 Solve for x:

$$3(x-1)^2 - 2(x-1) - 1 = 0.$$

6

(i) $3x^2 + 2x - 8$

(ii) $2x^2y - 2xz - 3xy + 3z$

(iii) $9a^2 - 12ab + 4b^2 - 16c^2$.

7

(a) Factorise fully each of the following:

(i) $6a^2 + 2ab + 3ac + bc$

(ii) $10x^2 - 3x - 1$

(iii) $5x^2 - 125y^2$.

(b) (i) Write the following as a single fraction:

$$\frac{3}{x+2} + \frac{6}{x-4}, \quad x \neq -2, x \neq 4.$$

(ii) Evaluate your answer when $x = 1$.

(c) (i) Solve, correct to one decimal place, the equation

$$5 - 3x - x^2 = 0.$$

7c When $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

-b formula

Bring Together 8

20 minute question

14% of 2017 Paper 1

(a) Factorise $n^2 - 11n + 18$.

When we factorise, we wish to put into bracket.

$n^2 - 11n + 18 = (n \quad)(n \quad)$ $\quad 11 = 9+2$!

So $(n-9)(n-2)$.

(b) Factorise fully $wy - y - 1 + w$.

Again we wish to put into brackets

w

$wy - y - 1 + w = wy - y + w - 1$

$= y(w-1) + (w-1)$

$= (y+1)(w-1)$.

(c) Find the value of $\frac{5}{3x-2} - \frac{7}{6x-12}$, when $x = 4$.

This is a tricky question we do not need to do anything other than substitute x with 4

So $\frac{5}{3(4)-2} - \frac{7}{6(4)-12} = \frac{5}{12-2} - \frac{7}{24-12} = \frac{5}{10} - \frac{7}{12}$

We need to find a common denominator (120) $\frac{60}{120} - \frac{70}{120} = -\frac{10}{120}$

So $-\frac{1}{12}$.

(d) Use factorisation to simplify $\frac{4e^2-9}{2e^2+3e-9}$.

Students find these difficult but usually the key to solution is in factorising the top or bottom (numerator or denominator) to give a hint to the factorisation

$4e^2 - 9 = (2e-3)(2e+3)$ difference of 2 squares (always comes up)

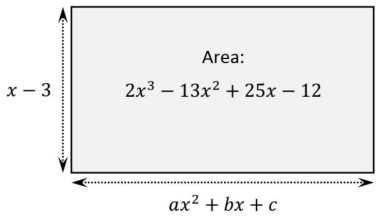
This tells you that the bottom will have one of those factors!

$2e^2 + 3e - 9 = (2e \quad)(e \quad)$

$-9 = -1 \times 9$ or -3×3, so by elimination we get $(2e-3)(e+3)$ ~ So can you see $\frac{e+3}{2e+3}$

(e) A rectangle has sides of length $x - 3$ units and $ax^2 + bx + c$ units, where $a, b, c \in \mathbb{Z}$.
The **area** of the rectangle is $2x^3 - 13x^2 + 25x - 12$ square units.

Find the value of a, the value of b, and the value of c.



This is long division in algebra. The area must be one side multiplied by the other:

$(x-3)(ax^2+bx+c) = 2x^3 - 13x^2 + 25x - 12$.

If so, I can divide $(x-3)$ into $2x^3 - 13x^2 + 25x - 12$ with no remainder?

$2x^2 - 7x + 4$

$x-3 \;|\; 2x^3 - 13x^2 + 25x - 12$

$2x^3 - 6x^2$

$-7x^2 + 25x$

$-7x^2 + 21x$

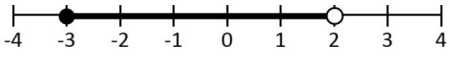
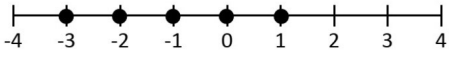
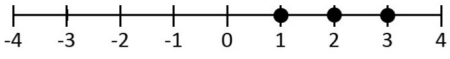
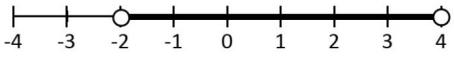
$4x - 12$

$4x - 12$

$0x + 0$

Can you now write down a, b, c?

9 Write down an inequality in x represented by each of the number lines shown below.
Put a tick (✓) in the correct box in each case to show whether $x \in \mathbb{N}$, $x \in \mathbb{Z}$, or $x \in \mathbb{R}$.
The first one is done.

Number line	Inequality in x	Domain (Tick **one** box only in each case)
	$-3 \leq x < 2$	☐ $\mathbb{N}$ ☐ $\mathbb{Z}$ ☑ $\mathbb{R}$
	$-3 \leq x \leq 1$	☐ $\mathbb{N}$ ☑ $\mathbb{Z}$ ☐ $\mathbb{R}$
	$1 \leq x \leq 3$	☑ $\mathbb{N}$ ☐ $\mathbb{Z}$ ☐ $\mathbb{R}$
	$-2 < x < 4$	☐ $\mathbb{N}$ ☐ $\mathbb{Z}$ ☑ $\mathbb{R}$

10 **(a)** Solve the following inequality and show the solution on the number line.

$$-2 \leq \frac{1}{2}x - 3 < 1, \quad x \in \mathbb{N}.$$

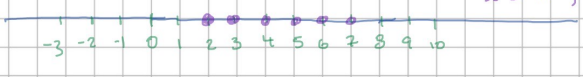
We can add, subtract (and multiply and divide by a positive number) and not change the inequality. The only thing we can't do is multiply or divide by a negative number.

We can take $-2 \leq \frac{1}{2}x - 3 < 1$ to

$1 \leq \frac{1}{2}x < 4$ to

$2 \leq x < 8$

$x \in \mathbb{N}$, the naturals.



(8 not included)

(b) Josephine hopes to go to college. She has saved €3000. She will attend college for 32 weeks in her first year. She plans to have at least €800 left at the end of the year.

(i) If she spends €x each week, write an inequality to represent her spending during the year.

She goes to college for 32 weeks. She has €3000, so she wants to have 800 left over AT LEAST!

$32x \leq 3000 - 800$

$32x \leq 2200$

If she waits anymore, she will have less than 800 euros left over.

(ii) Hence, or otherwise, find the maximum amount Josephine can spend each week.

$x = \frac{2200}{32} = €\ 68.75$

11 Solve the simultaneous equations:

$$3x + 4y = -1$$

$$2x + 9 = -6y.$$

12 Solve the simultaneous equations

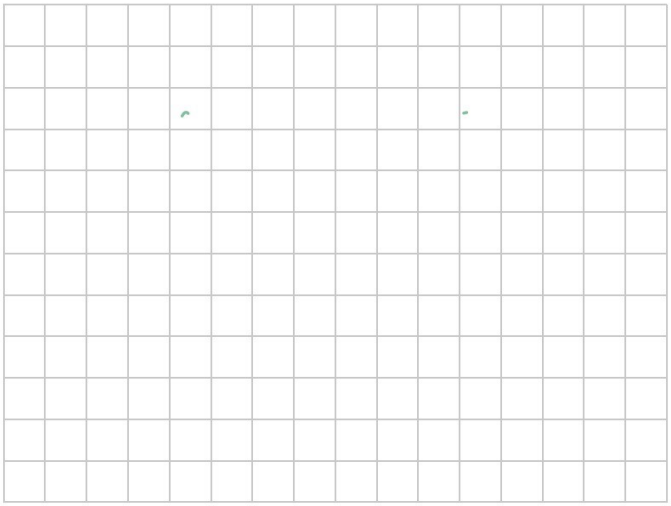
$$2x + 7y = 3$$

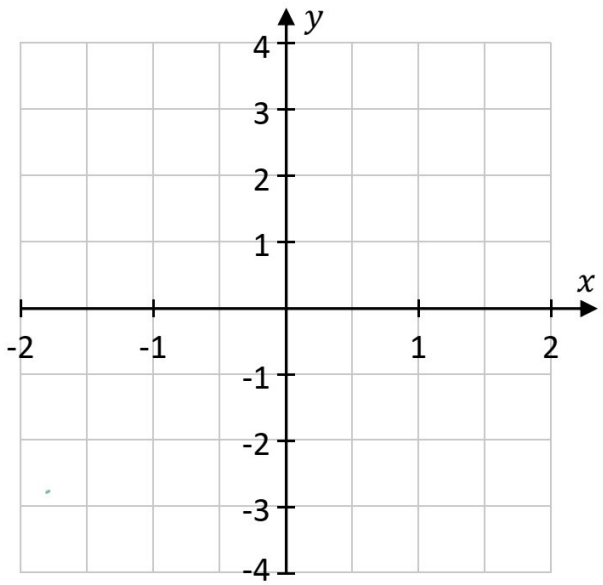
$$x + y = \frac{x - 2y + 1}{2}.$$

We need to be able to graph the following

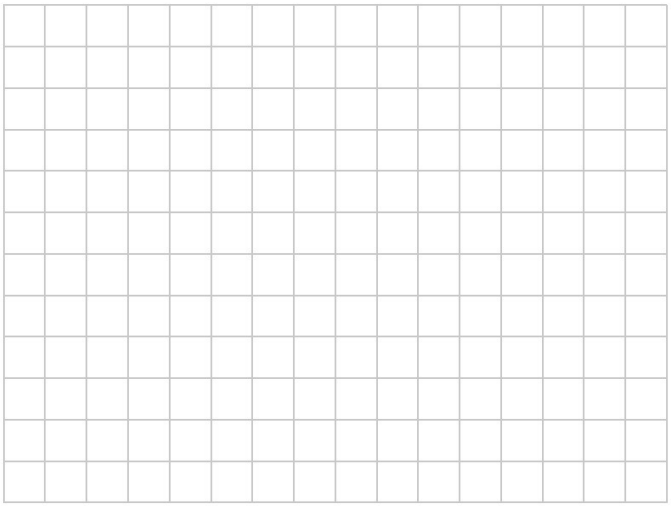
13 Graph each of the following three functions in the domain $-2 \leq x \leq 2$, for $x \in \mathbb{R}$.

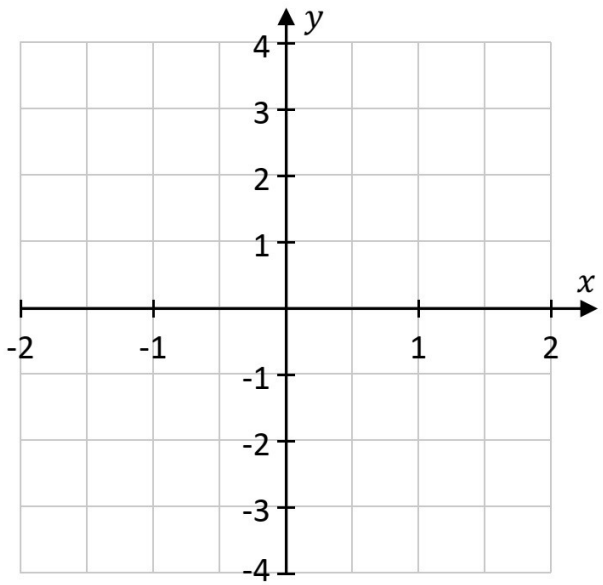
Function: $y = x - 1$



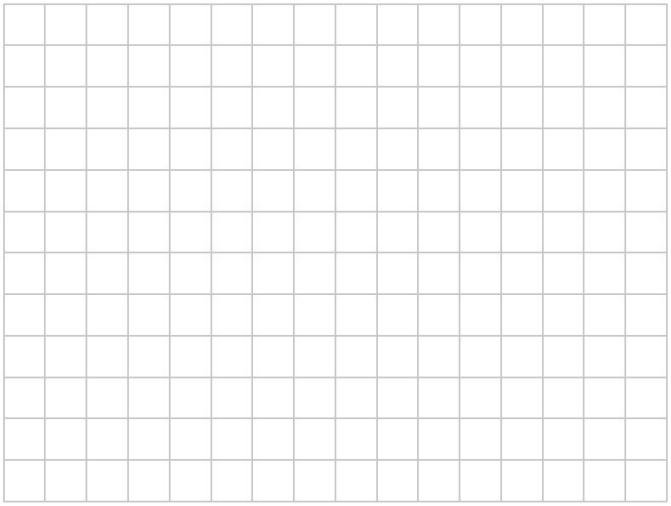


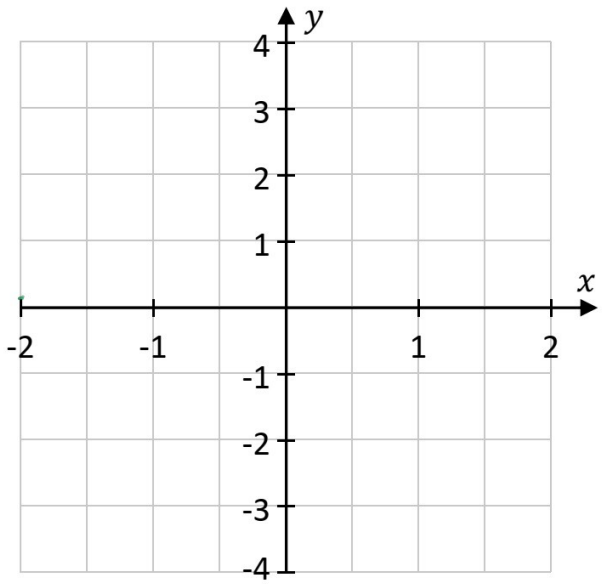
Function: $y = 2 - x^2$





Function: $y = 2^x$





14 **(a)** Factorise fully each of the following expressions.

(i) $9x^2 - 81$

(ii) $3x^2 - 17x + 10$

(iii) $4a - 6bc + 3ac - 8b$.

(b) **(i)** Write the following as a single fraction in its simplest form.

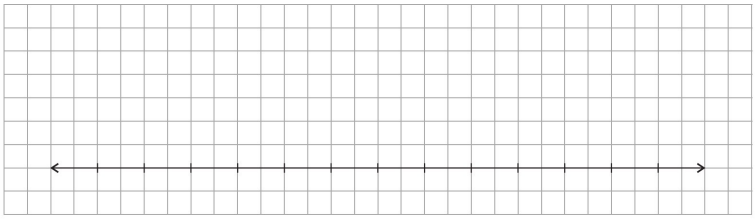
$$\frac{x+2}{3} - \frac{x-1}{2}$$

(ii) Hence, or otherwise, solve the equation

$$\frac{x+2}{3} - \frac{x-1}{2} = 6.$$

15 **(a)** Solve the following inequality and graph your solution on the number line.

$$-21 \leq 3 - 6x < 12, \quad x \in \mathbb{Z}.$$



(b) Factorise fully each of the following expressions.

(i) $27x^2 - 45x$

(ii) $6xy - 3sy - 4tx + 2st$

(c) A 99 Flake ice cream, more commonly known as a **99** or **ninety-nine**, is an ice cream cone in which a Flake bar is inserted.

Mary buys 2 plain ice cream cones and 3 ninety-nines costing €10·20.
Joe buys 3 plain ice cream cones and 4 ninety-nines costing €14·20.
Calculate the cost of a flake.

16 **(a)** Write down the smallest **prime number**. Give a reason for your answer.

Answer:

Reason:

(b) Explain why 8·7 is a **rational number**.

(c) Explain what is meant by an **integer number**.

(d) Write down an **irrational number** that lies between 3 and 4. Give a reason for your answer.

Answer:

Reason:

17 **(a)** $p = 2a + c$.

(i) Write a in terms of p and c.

(ii) Hence, find the value of a when $p = 5$ and $c = \frac{1}{5}$.

(b) Solve the equation $x^2 - 6x + 4 = 0$.
Give each answer in surd form.

(c) Write the following as a single fraction in its simplest form.

$$\frac{2x - 3}{2} - \frac{4x - 7}{5}.$$

18 **(a)** Factorise fully each of the following expressions.

(i) $15cd - 3de$

(ii) $om + np - no - pm$

(b) **(i)** Factorise $2x^2 - 5x - 12$.

(ii) Hence, or otherwise, solve the equation $2x^2 - 5x - 12 = 0$.

19 Transition year students are performing a school play.
Tickets cost €7 for adults and €3 for students and children.

On the opening night, they sold 299 tickets for a total of €1481.

(a) Write down two equations to represent the above information.

(b) How many adult tickets were sold?

(c) How many student and child tickets were sold?

20 **(a)** For each of the following, write down the value of x, where $21 \leq x \leq 27$ and $x \in \mathbb{N}$.

Description	Value of x
A square number	
A multiple of 7	
A factor of 66	
A cube number	
A prime number	

(b) $\sqrt{27} + \sqrt{75} - \sqrt{12} = k\sqrt{3}$, where $k \in \mathbb{Z}$.
Find the value of k.

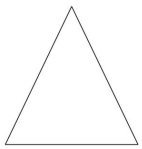
(c) **(i)** Simplify $(\sqrt[3]{x})^6$.

(ii) Simplify $\dfrac{3y^2 \times (10y)^3}{2y^5}$.

21 A triangle has three sides as follows

$3x + 2, \quad 2x - 5, \quad x + 1$

where $x \in \mathbb{R}$.



(a) Calculate the length of the perimeter of the triangle in terms of x.
Give your answer in its simplest form.

(b) Show that the triangle is isosceles when x is equal to 6.

(c) Mark says that x equal to 6 is the only value of x such that the triangle is isosceles.
Is he correct? Justify your answer.

Answer:

Justification: